Exercise 8G

1 a $y = 12x^2 + 3x + 8$

$\frac{dy}{dx} = 24x + 3$

$\frac{d^2y}{dx^2} = 24$

b $y = 15x + 6 + \frac{3}{x}$

$= 15x + 6 + 3x^{-1}$

$\frac{dy}{dx} = 15 - 3x^{-2}$

$\frac{d^2y}{dx^2} = 6x^{-3}$

c $y = 9\sqrt{x} - \frac{3}{x^2}$

$= 9x^{\frac{1}{2}} - 3x^{-2}$

$\frac{dy}{dx} = \frac{9}{2}x^{-\frac{1}{2}} + 6x^{-3}$

$\frac{d^2y}{dx^2} = -\frac{9}{4}x^{-\frac{3}{2}} - 18x^{-4}$

$= -\frac{9}{4\left(\sqrt{x}\right)^3} - \frac{18}{x^4}$

d $y = (5x + 4)(3x - 2)$

$= 15x^2 + 2x - 8$

$\frac{dy}{dx} = 30x + 2$

$\frac{d^2y}{dx^2} = 30$

e $y = \frac{3x + 8}{x^2}$

$= \frac{3x}{x^2} + \frac{8}{x^2}$

$= 3x^{-1} + 8x^{-2}$

$\frac{dy}{dx} = -3x^{-2} - 16x^{-3}$

$\frac{d^2y}{dx^2} = 6x^{-3} + 48x^{-4}$

2 $f(t) = \frac{t^2 + 2}{\sqrt{t}} = t^{\frac{3}{2}} + 2t^{-\frac{1}{2}}$

$f'(t) = \frac{3}{2}t^{\frac{1}{2}} - t^{-\frac{3}{2}}$

Acceleration $= f''(t) = \frac{3}{4\sqrt{t}} + \frac{3}{2\left(\sqrt{t}\right)^5}$

3 $y = (2x - 3)^3$

$= 8x^3 - 36x^2 + 54x - 27$

$\frac{dy}{dx} = 24x^2 - 72x + 54$

$\frac{d^2y}{dx^2} = 48x - 72$

$48x - 72 = 0$

$x = \frac{3}{2}$

4 $f(x) = px^3 - 3px^2 + x^2 - 4$

$f'(x) = 3px^2 - 6px + 2x$

$f''(x) = 6px - 6p + 2$

$f''(2) = -1$

$12p - 6p + 2 = -1$

$6p = -3$

$p = -\frac{1}{2}$